Induced Random Fields in the $LiHo_xY_{1-x}F_4$ Quantum Ising Magnet in a Transverse Magnetic Field

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The $LiHo_xY_{1-x}F_4$ magnetic material in a transverse magnetic field $B_x\hat{x}$ perpendicular to the Ising spin direction has long been used to study tunable quantum phase transitions in a random disordered system. We show that the B_x-induced magnetization along the $\hat{x}$ direction, combined with the local random dilution-induced destruction of crystalline symmetries, generates, via the predominant dipolar interactions between Ho^{3+} ions, *random fields* along the Ising $\hat{z}$ direction. This identifies $LiHo_xY_{1-x}F_4$ in B_x as a new random field Ising system. The random fields explain the rapid decrease of the critical temperature in the diluted ferromagnetic regime and the smearing of the nonlinear susceptibility at the spin-glass transition with increasing B_x and render the B_x-induced quantum criticality in $LiHo_xY_{1-x}F_4$ likely inaccessible.

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Quantum phase transitions (QPTs) occur near absolute zero temperature and are driven by quantum mechanical fluctuations associated with the Heisenberg uncertainty principle and not by thermal fluctuations as in classical phase transitions [1,2]. The transverse field Ising model (TFIM) [3,4] with Hamiltonian $\mathcal{H}_{\rm TFIM} = -\frac{1}{2}\sum_{(i,j)}J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_i\sigma_i^x$, where σ_i^μ ($\mu = x, y, z$) are Pauli matrices, is the simplest theoretical model that exhibits a QPT [1,4,5]. The field Γ transverse to the Ising $\hat{z}$ direction causes quantum tunneling between the spin-up and spin-down eigenstates of σ_i^z. These spin fluctuations decrease the critical temperature T_c at which the spins develop either conventional long-range order or, for random ferromagnetic and antiferromgnetic J_{ij}, a spin-glass (SG) state with randomly frozen spins below T_g. At a critical field Γ_c, T_c or T_g vanishes, and a quantum phase transition between either a long-range ordered or SG state and a quantum paramagnet (PM) ensues.

The phenomenology of both the disorder-free and the random TFIM has been extensively investigated in the $LiHo_xY_{1-x}F_4$ magnet with a magnetic field B_x applied transverse to the Ho^{3+} Ising spin direction [6–8], which is parallel to the c axis of the body-centered tetragonal structure of $LiHo_xY_{1-x}F_4$ [9]. Crystal field effects give an Ising ground state doublet $|\Phi_0^\pm\rangle$ and a first excited singlet $|\Phi_{\rm e}\rangle$ at approximately 9 K above the ground doublet [9]. For $x = 1$, $LiHoF_4$ is a dipolar-coupled ferromagnet (FM) with $T_c = 1.53$ K [7,10]. Random disorder is generated by replacing the magnetic Ho^{3+} ions by nonmagnetic Y^{3+}. Quantum mechanical (spin flip) fluctuations are induced by B_x which admixes $|\Phi_{\rm e}\rangle$ with $|\Phi_0^\pm\rangle$, splitting the latter, hence producing an effective TFIM with $\Gamma = \Gamma(B_x)$ ($\Gamma \propto B_x^2$ for small B_x) [10].

Two experimental puzzles pertaining to the effect of B_x on the FM ($0.25 < x < 1.0$) and the SG ($x < 0.25$) phases of $LiHo_xY_{1-x}F_4$ have long been known. First, in the FM regime, while the mean-field argument that $T_c(x) \propto x$ for the PM to FM transition is well satisfied for $0.25 < x < 1.0$ [8], the rate at which $T_c(B_x)$ is depressed by B_x becomes progressively faster than mean-field theory predicts as x is reduced [11]. This implies that, compared with the energy scale for FM order set by $T_c(B_x = 0)$, B_x becomes ever more efficient at destroying FM order the lower x is [11]. Second, for $B_x = 0$, $LiHo_{0.167}Y_{0.833}F_4$ displays a conventional SG transition, with a nonlinear magnetic susceptibility χ_3 diverging at T_g as $\chi_3(T) \propto (T - T_g)^{-\gamma}$ [12]. However, $\chi_3(T)$ becomes less singular as B_x is increased from $B_x = 0$, with no indication that a QPT between the PM and SG states occurs as $T \to 0$ [6,13]. It has recently been suggested that for dipole-coupled Ho^{3+} ions nonzero B_x generates both longitudinal (along the Ising $\hat{z}$ direction) [14] and transverse [15,16] random fields (RFs) that either renormalize the critical transverse field [14,15] or even destroy the SG transition [16]. In this Letter, we examine the quantitative merit of this hypothesis by comparing results from numerical and analytical calculations with experimental results on $LiHo_xY_{1-x}F_4$. We obtain compelling evidence that RFs are manifestly at play in $LiHo_xY_{1-x}F_4$ and explain the above two long-standing paradoxes.

We first show that the low-energy effective theory of $LiHo_xY_{1-x}F_4$ for $x < 1$ and $B_x > 0$ is a TFIM with additional B_x-induced RFs. We start with the Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm dip}$ expressed in terms of the angular momentum operator $\boldsymbol{J}$ of Ho^{3+} ($J = 8$, $L = 6$, $S = 2$). The single ion part $\mathcal{H}_0 = \sum_i[\mathcal{H}_{\rm cf}(\boldsymbol{J}_i) + \mathcal{H}_Z(\boldsymbol{J}_i)]$ consists of the crystal field Hamiltonian $\mathcal{H}_{\rm cf}(\boldsymbol{J}_i)$ of Ho^{3+} in the $LiHo_xY_{1-x}F_4$ environment [9,10] and the Zeeman field term $\mathcal{H}_Z = -g\mu_B(\boldsymbol{J}_i \cdot \boldsymbol{B})$, with $\boldsymbol{B}$ the magnetic field. $g = 5/4$ is the Ho^{3+} Landé factor, and μ_B is the

Bohr magneton. The interactions between ions are dominated by long-range magnetic dipolar interactions $\mathcal{H}_{\rm dip}$ [7,10,17], $\mathcal{H}_{\rm dip} = (g^2\mu_B^2/2)\sum_{(i,j)}\epsilon_i\epsilon_j[\boldsymbol{J}_i \cdot \boldsymbol{J}_j - 3(\boldsymbol{J}_i \cdot \boldsymbol{r}_{ij}\boldsymbol{J}_j \cdot \boldsymbol{r}_{ij})r_{ij}^{-2}]r_{ij}^{-3}$, where $\boldsymbol{r}_i$ are the crystalline positions occupied by either a magnetic Ho^{3+} ion ($\epsilon_i = 1$) or a nonmagnetic Y^{3+} ion ($\epsilon_i = 0$), and $r_{ij} = |\boldsymbol{r}_j - \boldsymbol{r}_i|$ is the inter-ion distance.

The two lowest energy eigenstates $|\Psi_i^+(B_x)\rangle$ and $|\Psi_i^-(B_x)\rangle$ of $\mathcal{H}_0$ are sufficiently below $|\Phi_e\rangle$ that the latter can be ignored at temperatures near and below $T_c(x = 1) \sim 1.5$ K [10]. This allows us to recast $\mathcal{H}$ in terms of an effective Hamiltonian $\mathcal{H}_{\rm eff}$ with $S = 1/2$ pseudospin operators that act in the restricted low-energy subspace spanned by the $\prod_i|\Psi_i^{\sigma_i}(B_x)\rangle$ ($\sigma_i = \pm$) eigenstates of $\mathcal{H}_0$. In this subspace, $\Gamma(B_x) \equiv (1/2)[\langle\Psi^+|\mathcal{H}_0|\Psi^+\rangle - \langle\Psi^-|\mathcal{H}_0|\Psi^-\rangle]$. The projected J_i^μ ($\mu = x, y, z$) operator is written as: $J_i^\mu = \sum_\nu C_{\mu\nu}(B_x)\sigma_i^\nu + C_{\mu 0}\mathbb{1}$ [10]. The $|+\rangle$ and $|-\rangle$ eigenstates of σ_i^z are written in terms of $|\Psi_i^\pm\rangle$ such that $J_i^z = C_{zz}\sigma_i^z$ [10]. For $B_x = 0$, only $C_{zz} \neq 0$ and decreases slightly with increasing B_x, while the other $C_{\mu\nu}(B_x)$ parameters and $\Gamma(B_x)$ increase with B_x, starting from zero at $B_x = 0$. By straightforward manipulations replacing J_i^μ in terms of $\sum_\nu C_{\mu\nu}\sigma_i^\nu$ in $\mathcal{H}_{\rm dip}$, one finds that the terms with largest $C_{\mu\nu}(B_x)$ in $\mathcal{H}_{\rm eff}$, including the transverse field term $\Gamma\sigma_i^x$, are

$$\mathcal{H}_{\rm eff} = \frac{(g\mu_B)^2}{2}C_{zz}^2\sum_{(i,j)}\epsilon_i\epsilon_j L_{ij}^{zz}\sigma_i^z\sigma_j^z - \Gamma\sum_j\sigma_i^x + (g\mu_B)^2C_{zz}\Big\{C_{x0}\sum_{(i,j)}\epsilon_i\epsilon_j L_{ij}^{xz}\sigma_i^z + C_{xx}\sum_{(i,j)}\epsilon_i\epsilon_j L_{ij}^{xz}\sigma_i^x\sigma_i^z\Big\}, \tag{1}$$

where $L_{ij}^{ab} = [\delta_{ab} - 3r_{ij}^a r_{ij}^b/r_{ij}^2]r_{ij}^{-3}$. We see from Eq. (1) that a longitudinal RF term $\propto \sigma_i^z$ along $\hat{z}$ and an off-diagonal (bilinear) $\propto \sigma_i^x\sigma_j^z$ coupling are induced by $B_x \neq 0$ ($C_{x0} = C_{xx} = 0$ for $B_x = 0$). For pure $LiHoF_4$ ($x = 1$, all $\epsilon_i = 1$), lattice symmetries enforce $\sum_j L_{ij}^{xz} = 0$, causing the term linear in σ_i^z and the Boltzmann thermal average $\langle\sigma_i^x\sigma_j^z\rangle$ to vanish for $B_x > 0$. However, for $x < 1$ and $B_x > 0$, a RF $\propto \sigma_i^z$ emerges. With the time-reversal symmetry broken by B_x ($\langle J_i^x\rangle > 0$), the bilinear $\sigma_i^x\sigma_j^z$ also provides a "mean-field" contribution to the longitudinal RFs, $\langle\sigma_i^x\rangle\sigma_j^z$, as well as transverse RFs, $\langle\sigma_i^z\rangle\sigma_j^x$. We find that $C_{xx}/C_{x0} \lesssim 1$ for all $B_x > 0$ so that the leading correction to $H_{\rm TFIM}$ is indeed a (correlated) RF term, $\sim\sum_i h_i^z\sigma_i^z$, with $h_i^z \propto C_{x0}C_{zz}\sum_j\epsilon_j L_{ij}^{xz}$.

We now investigate the effect of the RFs on the PM to FM transition. To do so, we make a mean-field (MF) approximation to $\mathcal{H}$ and consider the one-particle MF Hamiltonian $\mathcal{H}_i^{\rm MF}$ for an arbitrary site $\boldsymbol{r}_i$ occupied by a Ho^{3+} moment: $\mathcal{H}_i^{\rm MF} = \mathcal{H}_{\rm cf} - g\mu_B(\boldsymbol{h}_i^{\rm MF} \cdot \boldsymbol{J}_i) - g\mu_B J_i^x B_x$. $\boldsymbol{h}_i^{\rm MF}$ is the MF acting on magnetic moment $\boldsymbol{J}_i$, $\boldsymbol{h}_i^{\rm MF} = g\mu_B\sum_j\epsilon_j[3(\boldsymbol{r}_{ij} \cdot \boldsymbol{M}_j)\boldsymbol{r}_{ij}/r_{ij}^2 - \boldsymbol{M}_j]r_{ij}^{-3}$, with the self-consistent MF equation $\boldsymbol{M}_i = \langle\boldsymbol{J}_i\rangle$, with $\boldsymbol{M}_i = {\rm Tr}(\rho_{\rm MF}\boldsymbol{J}_i)/{\rm Tr}(\rho_{\rm MF})$ and $\rho_{\rm MF} = \exp(-\beta H_i^{\rm MF})$, with $\beta \equiv 1/(k_BT)$. The crystal field Hamiltonian $H_{\rm cf}$ for Ho^{3+} expressed in terms of the components J_i^μ is taken from the 4 crystal field parameter $\mathcal{H}_{\rm cf}$ of Ref. [9]. Slightly different choices of $\mathcal{H}_{\rm cf}$ [9,10] do not qualitatively affect the results. We diagonalize $\mathcal{H}_i^{\rm MF}$ for each i and update the $\boldsymbol{M}_i$'s using the newly obtained eigenstates. We continue iterating until convergence is reached at the nth iteration, defined by the convergence criterion $\sum_i(\boldsymbol{M}_i^{(n)} - \boldsymbol{M}_i^{(n-1)})^2/\sum_i(\boldsymbol{M}_i^{(n)})^2 \leq 10^{-6}$. To simplify the calculations and speed up the convergence, we keep only the diagonal L_{ij}^{zz} and the off-diagonal L_{ij}^{xz} terms in $\mathcal{H}_{\rm dip}$, since no qualitatively new physics arises when keeping the other diagonal (L_{ij}^{xx} and L_{ij}^{yy}) and off-diagonal (L_{ij}^{xy}, L_{ij}^{yz}) terms in $\mathcal{H}_{\rm dip}$. For a given B_x, we compute the temperature dependence of the magnetization $M_z = [\sum_i M_i^z]_d$ along the $\hat{z}$ direction [18]. Here $[\ldots]_d$ signifies an average over the bimodal lattice occupancy probability distribution $P(\epsilon_i) = x\delta(\epsilon_i - 1) + (1 - x)\delta(\epsilon_i)$. The transition temperature $T_c(B_x)$ is determined by the temperature at which $M_z(T)$ sharply rises [18]. We consider a system of linear size $L = 4$, with the total number of sites $N_0 = 4L^3$ with $N = xN_0$ sites occupied by Ho^{3+}, and perform disorder averages over 50 different diluted samples. We use the Ewald summation method to define infinite-range dipolar interactions. For $B_x = 0$, we find that, as found experimentally, the decrease of T_c while reducing x is proportional to x [8,11]. To investigate the role of RFs, we compare $T_c(B_x)$ obtained when both the L_{ij}^{xz} and L_{ij}^{zz} terms in $\mathcal{H}_{\rm dip}$ are kept (open symbols) with the $T_c(B_x)$ found when only the L_{ij}^{zz} term is retained (solid symbols). Figure 1 shows that $T_c(B_x)$ is depressed for small B_x faster when the off-diagonal L_{ij}^{xz} term in $\mathcal{H}_{\rm dip}$ is present than when only the

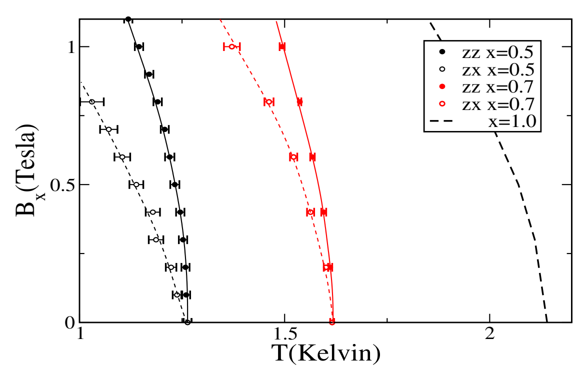

FIG. 1 (color online). $T_c(B_x)$ from a finite lattice mean-field calculation (see text). For $x = 0.7$ and $x = 0.5$, the solid symbols show $T_c(B_x)$ when only the L_{ij}^{zz} dipolar terms that couple z components of $\boldsymbol{J}_i$ are kept. The open symbols show the increased rate of depression of $T_c(B_x)$ when the off-diagonal dipolar L_{ij}^{xz} couplings are included. For the pure $x = 1$ case, $T_c(B_x)$ is the same for both models since the lattice symmetries eliminate the internal local random fields along $\hat{z}$.

Ising L_{ij}^{zz} term is kept. Also, as found experimentally [11], the rate at which $T_c(B_x)$ is depressed by B_x increases as x is lowered. This provides strong evidence that the experimental observation is due to B_x-induced RFs whose variance increases as x decreases or B_x increases.

We now consider the role of RFs at the SG transition. While the nonlinear susceptibility χ_3 diverges at $T_g \simeq 0.13$ K in $LiHo_{0.167}Y_{0.833}F_4$ when $B_x = 0$, $\chi_3(B_x, T)$ becomes progressively smeared as B_x is turned on (see top inset in Fig. 3) [6,13]. It has been suggested that random off-diagonal dipolar couplings destroy the SG transition when $B_x > 0$ [16]. Here we take a more pragmatic approach and ask whether the behavior of $\chi_3(B_x > 0, T)$ in SG samples of $LiHo_xY_{1-x}F_4$ can indeed be interpreted in terms of induced RFs. To investigate this question on $\chi_3(B_x, T)$, we introduce a mean-field variant of $\mathcal{H}_{\rm eff}$ in Eq. (1) that preserves the crucial physics therein. Our model is a generalization of the Sherrington-Kirkpatrick transverse field Ising SG model [5,19–21] but with additional off-diagonal and RF interactions similar to that in $\mathcal{H}_{\rm eff}$:

$$\tilde{\mathcal{H}} = \frac{1}{2}\sum_{(i,j)} J_{ij}\sigma_i^z\sigma_j^z + \frac{1}{2}\sum_{(i,j)} K_{ij}\sigma_i^x\sigma_j^z - \Gamma\sum_i \sigma_i^x - \sum_i h_i^z\sigma_i^z - h_0^z\sum_i \sigma_i^z. \quad (2)$$

For simplicity, we take infinite-ranged J_{ij} and K_{ij} given by independent Gaussian distributions of zero mean and variance J^2 and K^2, respectively. h_i^z is a Gaussian RF with zero mean and variance Δ^2. Model (2), but with $K_{ij} = 0$, was previously used to calculate χ_3 in quadrupolar glasses [19], which also possess internal RFs [19,22].

We follow the procedure of Ref. [21], employing the imaginary time formalism and the replica trick to derive the (replicated) free energy of the system. To further simplify the calculations, we make a static approximation for the replica-symmetric solution in the PM phase. This allows us to derive self-consistent equations for the α components of the magnetization M_α and spin-glass order parameters Q_α ($\alpha = x, z$). We find $M_\alpha = (1/2\pi)\int_{-\infty}^{\infty} dx dz e^{-(x^2+z^2)/2}[(H_\alpha/H)\tanh(\beta H)]^p$, with $p = 1$, $H_z = [h_0^z + z\sqrt{J^2Q_z + (K^2/2)Q_x + \Delta^2}]$, $H_x = [\Gamma + x\sqrt{(K^2/2)Q_z}]$, and $H^2 = H_x^2 + H_z^2$. The self-consistent equations for Q_x and Q_z are obtained by replacing $M_\alpha \to Q_\alpha$ above and setting $p = 2$. χ_3 is obtained from $\chi_3 = \frac{1}{6}\partial^3 M_z/(\partial h_0^z)^3$ and by solving numerically the resulting four coupled self-consistent equations. Figures 2(a) and 2(b) show the Γ dependence of χ_3 for various temperatures T in models either with only h_i^z random fields [Fig. 2(a)] or with only random off-diagonal K_{ij} couplings [Fig. 2(b)].

Comparing Fig. 2 with the experimental $\chi_3^{\rm expt}(B_x)$ (top inset in Fig. 3), one finds that the dependence of χ_3 upon Γ in Fig. 2(a) does not show a decrease in magnitude as T is

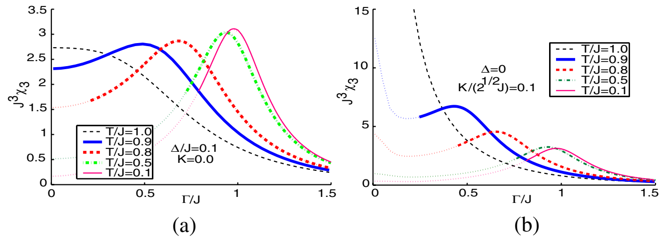

FIG. 2 (color online). (a) χ_3 vs Γ with $K = 0$. (b) χ_3 vs Γ with $\Delta = 0$. The change of line style at low Γ indicates the limit of stability of the replica-symmetric paramagnetic solution, as determined following the standard procedure [21].

decreased. Also, while χ_3 shows a decreasing amplitude with decreasing T in Fig. 2(b), it does not reveal a rapid sharpening as T is increased. It turns out that the key physics ingredient missing in these calculations is the underlying *microscopic* dependence of J, Δ, and K upon B_x via the $C_{\mu\nu}(B_x)$ transformation coefficients. Physically, the built-in B_x dependence of the $C_{\mu\nu}$ ensures that the B_x scale at which χ_3 is quenched by RFs is not trivially tied to the scale at which the $T = 0$ SG-PM crossover occurs, as it is in $\tilde{\mathcal{H}}$ with J_{ij}, K_{ij}, Γ, and h_i^z independent of B_x (cf. Fig. 2). The widths $J(B_x)$, $\Delta(B_x)$, and $K(B_x)$ are obtained by calculating the disorder average of the first, third, and fourth lattice sums in Eq. (1), respectively. We have $K^2 = 4(g\mu_B)^4[C_{zz}(B_x)C_{xx}(B_x)]^2 \times [(1/N_0)\sum_{(i,j)}\epsilon_i\epsilon_j(L_{ij}^{xz})^2]_d$ and $\Delta^2 = (g\mu_B)^4 \times [C_{zz}(B_x)C_{x0}(B_x)]^2[(1/N_0)\sum_i\epsilon_i(\sum_{j\neq i}\epsilon_j L_{ij}^{xz})^2]_d$. To make further contact between calculations and experimental data, we note that hyperfine interactions, which are important in Ho-based materials [7,10], lead to a renormalization of the critical B_x, B_x^c, when T_c or T_g is less than the hyperfine energy scale [10,15]. To obtain a relation be-

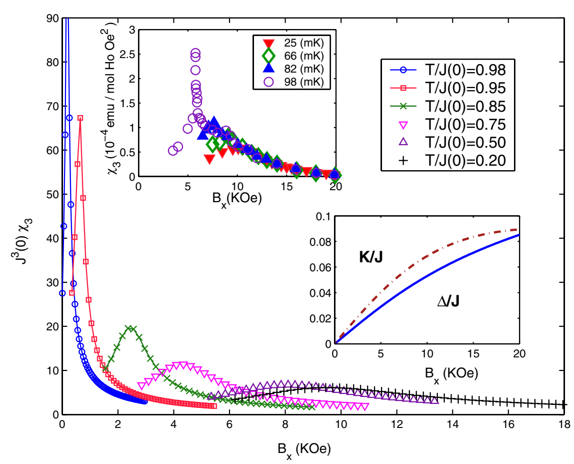

FIG. 3 (color online). $\chi_3^{\rm theor}$ vs B_x for different temperatures given by the model Eq. (2). The top inset shows $\chi_3^{\rm expt}$ from Ref. [6]. The parameters K and Δ are computed using $x = 0.167$ and rescaled by $\eta = 0.15$ and are shown in the bottom inset (see text).

tween Γ and B_x in $\tilde{\mathcal{H}}$ which does not incorporate hyperfine effects, we set $\Gamma(B_x)/J(B_x) = 1.05(B_x/B_x^c)^{0.35}$, where $B_x^c = 1.2$ T is the experimental zero temperature critical field [13]. This ansatz for $\Gamma(B_x)$ is obtained by matching the critical temperature $T_g(B_x)$ of $LiHo_xY_{1-x}F_4$ with the $T_g(\Gamma)$ of $\tilde{\mathcal{H}}$. For the former, we use $T_g(B_x) = T_g(0) \times [1 - (B_x/B_x^c)^\phi]$ ($\phi \approx 1.7$) as found experimentally [13]. For $\tilde{\mathcal{H}}$, we find $T_g(\Gamma) \approx T_g(0)\{1 - a[\Gamma/T_g(0)]^\psi\}$ ($a \approx 0.79$ and $\psi \approx 4.82$) by fitting $T_g(\Gamma)$ vs Γ with $K = \Delta = 0$. To incorporate the role of hyperfine effects on h_i^z and K_{ij}, we rescale K and Δ calculated from their microscopic origin in Eq. (1) by a scale factor $\eta = 0.15$ that positions the peak of $\chi_3^{\mathrm{theor}}(B_x)$ at $B_x \sim 5$ kOe for $T_g(B_x)/T_g(0) = 0.75$. With an estimate of η available, one could then determine a parametric $\Gamma(B_x)$ such that, for a given $T/T_g(B_x = 0)$, the experimental χ_3^{expt} and the theoretical χ_3^{theor} peak at the same B_x value for all $T_g(B_x)/T_g(0)$. Such a procedure gives results qualitatively very similar to those in Fig. 3.

χ_3^{theor} reproduces the overall trend of χ_3^{expt}. This is evidence that RFs are indeed at play when $B_x > 0$ and at the origin of the experimental $\chi_3(B_x, T)$ behaviors in SG samples of $LiHo_xY_{1-x}F_4$ [6,13]. A noticeable difference between χ_3^{theor} and χ_3^{expt} is that, for fixed $T/T_g(0)$, χ_3^{theor} collapses more rapidly and at smaller B_x than χ_3^{expt}. This is likely a further manifestation of the renormalization effects of the RFs and random off-diagonal couplings caused by the aforementioned hyperfine interactions. We will report elsewhere results addressing this hyperfine renormalization of $\mathcal{H}_{\mathrm{eff}}$ and its role on $\chi_3(B_x, T)$.

In conclusion, by comparing numerical and analytical results with experimental data, we have obtained compelling evidence that induced RFs are indeed at play and "observed" in $LiHo_xY_{1-x}F_4$. As a result, $LiHo_xY_{1-x}F_4$ in a transverse field (TF) is identified as a new RF Ising system. As found in other RF systems, we expect that the nontrivial fixed point of the theory ($\mathcal{H}_{\mathrm{eff}}$) is controlled by a fluctuationless *classical* zero temperature fixed point. In particular, this is what occurs in a TFIM plus RFs ($\tilde{\mathcal{H}}$ with $K_{ij} = 0$) [23]. Hence, it would therefore seem that quantum criticality is most likely innaccessible in ferromagnetic $LiHo_xY_{1-x}F_4$ samples. For the Ising SG model with RFs along $\hat{z}$, recent studies suggest that there is no Almeida-Thouless line [12] and no thermodynamic SG transition [24]. Hence, from the arguments above leading to $\mathcal{H}_{\mathrm{eff}}$, B_x-induced SG to PM quantum criticality in $LiHo_xY_{1-x}F_4$ would also appear likely inexistent. The presence of B_x-induced RFs and the quenching of quantum criticality is presumably the reason why, unlike in quantum Monte Carlo simulations of TF Ising SG models [20], Griffiths-McCoy singularities [25] have not been reported in $LiHo_xY_{1-x}F_4$ [6,8]. While quantum criticality seems unlikely for *any* $B_x > 0$ and $x < 1$, a new set of interesting questions has nevertheless arisen: Do FM samples of $LiHo_xY_{1-x}F_4$ with $B_x > 0$ indeed display classical RF criticality and all the fascinating phenomena of the RF Ising model [26]? Given the scarcity of real RF Ising materials [26], the identification of another such system opens new avenues for future theoretical and experimental investigations.

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